Student Workbook

Counting Principles

Name: ______________________ Date: ______________ Class: ________

ESSENTIAL QUESTION

How can we systematically count the number of ways events can occur without listing every possibility?

Lesson Overview

Counting principles give us powerful shortcuts for determining how many outcomes are possible in multi-step processes. The Fundamental Counting Principle says: if event A can occur in m ways and event B can occur in n ways, then both events together can occur in m × n ways. Permutations count arrangements where order matters; combinations count selections where order does not matter.

$$P(n,r) = n!/(n-r)!$$
$$C(n,r) = n!/[r!(n-r)!]$$
$$n^r \text{ (with repetition)}$$

Factorial (n!)	The product of all positive integers from 1 to n; 0! = 1 by definition.
Permutation P(n,r)	An arrangement where order matters: $P(n,r) = n! / (n-r)!$
Combination C(n,r)	A selection where order does not matter: $C(n,r) = n! / [r!(n-r)!]$
Repetition Allowed	Each position can reuse any value; total outcomes = n^r.

Worked Examples

EXAMPLE 1

A restaurant has 4 appetizers, 6 entrees, and 3 desserts. How many different 3-course meals are possible?

- Identify the choices at each stage: 4 appetizers, 6 entrees, 3 desserts.
- Apply the Fundamental Counting Principle: 4 × 6 × 3

Answer: There are 72 different 3-course meals.

EXAMPLE 2

How many ways can 5 different books be arranged on a shelf?

- All 5 books are placed in a row — order matters, no repetition.
- This is a permutation of 5 objects taken 5 at a time: $P(5,5) = 5! = 5×4×3×2×1$

Answer: There are 120 ways to arrange 5 books.

EXAMPLE 3

How many ways can a president, vice-president, and secretary be chosen from a group of 10 people?

- Order matters (different offices = different roles), no repetition.
- Use $P(n,r)$ with $n = 10$, $r = 3$: $P(10,3) = 10!/(10-3)! = 10 \times 9 \times 8$

Answer: There are 720 ways to choose the three officers.

EXAMPLE 4

How many ways can a committee of 4 be chosen from 9 people?

- A committee has no ranked positions — order does not matter.
- Use $C(n,r)$ with $n = 9$, $r = 4$: $C(9,4) = (9\times8\times7\times6)/(4\times3\times2\times1) = 3024/24$

Answer: There are 126 ways to form the committee.

EXAMPLE 5

A 4-digit PIN uses digits 0–9 with repetition allowed. How many different PINs are possible?

- Each of the 4 positions can be any of 10 digits (0–9), repetition allowed.
- Use the repetition formula: $n^r = 10^4$

Answer: There are 10,000 possible 4-digit PINs.

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 A coin is flipped 3 times. How many total outcomes are possible?

Hint: Each flip has 2 outcomes. Use the Fundamental Counting Principle: 2×2×2.

2 How many ways can the letters in the word MATH be arranged?

Hint: All 4 letters are distinct and order matters. Compute 4! = 4×3×2×1.

3 How many ways can 1st, 2nd, and 3rd place be awarded among 8 runners? Use $P(8,3)$.

Hint: $P(8,3) = 8!/(8-3)! = 8!/5! = 8\times7\times6$. Multiply those three numbers.

4 How many ways can 2 toppings be chosen from 8 available toppings? Use $C(8,2)$.

Hint: $C(8,2) = 8!/[2!\times6!] = (8\times7)/(2\times1)$. Order doesn't matter — it's a combination.

5 A password consists of 3 letters (A–Z, no repeat) followed by 2 digits (0–9, no repeat). How many passwords are possible?

Hint: Letters: P(26,3) = 26×25×24. Digits: P(10,2) = 10×9. Multiply the two results together.

Key Vocabulary

Fundamental Counting Principle	If event A has m outcomes and event B has n outcomes, both together have m × n outcomes.
Factorial (n!)	The product of all positive integers from 1 to n; 0! = 1.
Permutation	An arrangement of objects where order matters. P(n,r) = n!/(n−r)!
Combination	A selection of objects where order does not matter. C(n,r) = n!/[r!(n−r)!]
Repetition allowed	Each position can use any value again; total outcomes = n^r.

Quick Check Quiz

Circle the letter of the correct answer.

1 **A diner offers 3 soups and 4 sandwiches. How many soup-and-sandwich combos are there?** Multiple Choice

A 7
B 12
C 24
D 1

Common Mistakes to Avoid

✗ Using a permutation when choosing a committee (order doesn't matter for committees).

✓ Use C(n,r) for committees, teams, or any selection where roles are identical.

✗ Forgetting that 0! = 1, which causes errors in permutation/combination formulas.

✓ Always remember 0! = 1 by definition.

✗ Using n^r when repetition is NOT allowed.

✓ n^r only applies when repetition is allowed. Without repetition, use P(n,r) or C(n,r).

✗ Canceling factorials incorrectly, e.g., writing 10!/7! = 3!

✓ 10!/7! = 10 × 9 × 8 (cancel the 7! from both numerator and denominator).

Math Tips

★ Ask yourself: 'Does the order of selection matter?' Yes → permutation. No → combination.

★ $C(n,r) = C(n, n-r)$: choosing 4 from 10 gives the same count as choosing 6 from 10.

★ For multi-stage problems, multiply the counts at each stage (Fundamental Counting Principle).

★ Simplify factorials before multiplying: $P(10,3) = 10 \times 9 \times 8$, not the full 10!.

★ Passwords and PINs with repetition → use n^r. Without repetition → use $P(n,r)$.